Int. J. Heat Mass Transfer. Vol. 18, pp. 139–149. Pergamon Press 1975. Printed in Great Britain

PRESSURE LOSSES AND HEAT TRANSFER IN NON-CIRCULAR CHANNELS WITH HYDRAULICALLY SMOOTH WALLS

J. MALÁK, J. HEJNA and J. SCHMID
The Nuclear Research Institute, Řež near Praha, Czechoslovakia

(*Received* 28 *June* 1973)

Abstract—The article deals with the determination of the effect of channel geometry on friction pressure losses and heat transfer in the developed turbulent flow in non-circular channels with hydraulically smooth walls. Universal dimensionless relationship for friction coefficient is formulated which takes into account the geometry effect expressed in terms of the geometry factor κ_T. The empirical relationship for the geometry factor κ_T and channel shape is evaluated by two ways: by the integral geometry criterion L^* and by the laminar geometry factor κ_L.

Using an analogy between momentum and heat transfer the universal dimensionless relationship for the Nusselt number at isothermal flow is deduced from the universal criterion relationship for ξ.

An extensive experimental material was treated to evaluate and test these two universal relationships. In most cases compared an excellent agreement between experimental results and the universal criterion relationship was found.

NOMENCLATURE

c_p, specific heat at constant pressure [J.kg^{-1}.deg^{-1}];
C, constant in equation (18);
d, tube diameter, rod diameter [m];
d_1, d_2, inner and outer diameter of annulus [m];
d_h, $= \frac{4F}{S}$, hydraulic diameter [m];
d_0, characteristic dimension of channel cross-section [m];
d_T, defined at the equation (27) [m];
F, cross-sectional area [m^2];
$L(M_0)$, defined by equation (4) [m];
$\bar{L}$, mean value of L in cross section [m];
L^*, integral geometrical criterion;
Nu, $= \frac{\bar{\alpha}.d_h}{\lambda}$ Nusselt number;
p, static pressure [N.m^{-2}];
Pr, Prandtl number;
Re, $= \frac{\bar{w}.d_h}{\nu}$ Reynolds number;
S, wetted perimeter [m];
S_T, heated perimeter [m];
s, rod center distance [m];
v^*, wall shear velocity [m.s^{-1}];
$\bar{v}^*$, mean value of v^* on the wetted perimeter [m.s^{-1}];
$\bar{w}$, mean value of w in channel cross-section [m.s^{-1}];
w^+, inner-law velocity;
x, y, z, coordinates [m];
y, distance from wall along normal to wetted perimeter [m];
y^+, inner-law dimensionless distance from wall.

Greek symbols

α, heat-transfer coefficient [W.m^{-2}.deg^{-1}];
$\bar{\alpha}$, mean value of α on heated perimeter [W.m^{-2}.deg^{-1}];
ε, excentricity;
ε_T, eddy conductivity [m^2.s^{-1}];
κ_L, laminar geometrical factor;
κ_T, turbulent geometrical factor;
λ, thermal conductivity [W.m^{-1}.deg^{-1}];
μ, ν, molecular and kinematic viscosity [N.s.m^{-2}; m^2.s^{-1}];
ξ, friction factor (Blasius);
ξ_0, friction factor for smooth tube;
τ, wall shear stress [N.m^{-2}];
ρ, density of fluid [kg.m^{-3}];
$\bar{\tau}$, mean value of τ on wetted perimeter [N.m^{-2}];
$\bar{\tau}_T$, mean value of τ on heated perimeter [N.m^{-2}];
τ^*, $= \frac{\tau}{\bar{\tau}}$, relative wall shear stress;
ϕ, defined by equation (24);
ψ, temperature factor.

INTRODUCTION

THE KNOWLEDGE concerning pressure losses and heat transfer in the non-circular channels at a developed turbulent flow is based up to present practically only on empirical knowledge the experiments being essential means for its extension. Calculation methods elaborated in recent years by far do not permit to reply with a necessary accuracy on the question on the pressure losses and heat transfer in a channel which has not been previously examined experimentally. This is due to the fact that empirical information used in these methods as initial assumptions (e.g. the velocity distribution along the normals to the wetted perimeter and the distribution of wall shear stresses) may have limited validity and their generalization and application to different channel geometries may lead to incorrect results.

The situation in this field is even so bad that the knowledge of each particular geometry represents an isolated point in the steady growing experimental material. The whole region of geometries that is very important for practice thus becomes under this situation a set of individual relationships some basic similarity relations between them, however, being absent. In addition the experimental material, which is already relatively extensive at present, has a fundamental shortcoming. By comparing the measurement results of different authors on the channels of the same geometry non-negligible differences appear between their results and sometimes even contradictions exist. The channel in the form of a smooth annulus may be named as a typical case. Though one of the simplest geometries is involved great differences exist between the criterion relationships for the friction factor and Nusselt number by different author. These differences attain even several tens of percent (for detailed comparisons see [1–3]).

A number of reasons of these differences may exist, an appropriate evaluation of all factors intervening in the results, however, is not possible without detailed knowledge of experimental conditions and analysis of basic experimental results. We are of the opinion, however, that the main misleading factors are those which are essentially connected with the process investigated their misleading effect being a consequence of insufficient knowledge of the process studied in non-circular channels or a consequence of an inconvenient application of the knowledge of flow conditions in smooth tubes in non-circular channels.

This applies especially for such notions as follows:

1. Position and range of the transition region of flow and their dependence on inlet conditions.
2. Length of the hydrodynamical entrance section.
3. Length of the temperature entrance section.
4. Effect of wall roughness.
5. Effect of non-isothermal flow on heat transfer.

These factors may have rather different quantitative and qualitative effect in non-circular channels and in the channels of circular cross-section. A more detailed analysis of these factors and some comparison of the conditions in circular and non-circular channels are carried out in [1].

The introduction of the hydraulical diameter as a characteristic channel dimension together with the assumption of the general validity of the relations determined initially for a circular channel may be considered as the first attempt to generalize experimental information. It is well known that the applicability of this method is limited towards the channels of more complex geometries. We have tried, therefore, to find new criterion relationships for the calculation of pressure losses and heat transfer in non-circular channels which would take into account the effect of geometry channel in a more adequate manner. We started here from a rich experimental material which has been obtained in the Nuclear Research Institute by the experimental research of pressure losses and heat transfer in channels having cross sections in the form of a smooth annulus, a longitudinally finned annulus (denoted as "A" geometry), and tubes with inner longitudinal fins (denoted as "B" geometry). Totally sixty strongly differing (with respect to geometry) channels were investigated [4, 5]. Detailed knowledge of experimental conditions together with a great amount of experimental values served as a fundamental basis for such attempt.

THE FRICTION FACTOR

It has been found by a detailed analysis of experimental results from pressure loss measurements that in all cases in which the perturbing effect mentioned above do not appear the friction factor for developed turbulent flow satisfies the relationship

$$\frac{1}{\sqrt{\xi}} = C_1 \log Re\sqrt{\xi} + C_2 \qquad (1)$$

the constants C_1 and C_2 being only a function of channel geometry. The friction factor is defined here by the equation

$$\xi = 2d_h \left|\frac{\mathrm{d}p}{\mathrm{d}z}\right| \left(\frac{1}{\rho \bar{w}^2} - \frac{1}{p}\right),$$

where d_h is the hydraulic diameter and $\bar{w}$ mean velocity.

It has been shown by further analysis that the constants C_1 and C_2 are mutually dependent and that the general relationship (1) may be expressed by a simple transformation with an accuracy corresponding to the accuracy of the experiments by the well known Prandtl law of friction in a smooth tube. This transformation corresponds to a substitution of the hydraulic diameter with a more convenient characteristic dimension. Denoting this dimension by d_0 and by subscript "0" the quantities related to this dimension then the transformation mentioned may be described by following equations

$$\frac{\xi}{\xi_0} = \frac{Re}{Re_0} = \frac{d_h}{d_0} = \kappa_T \tag{2}$$

where κ_T is the geometrical factor for turbulent flow. The universal criterion relationship is then of the form

$$\sqrt{\left(\frac{\kappa_T}{\xi}\right)} = 2\log\frac{Re\sqrt{\xi}}{\kappa_T^{3/2}} - 0{\cdot}8. \tag{3}$$

This fundamental result reduces the problem of finding the criterion relationship for ξ to the question on which geometrical quantity depends the geometrical factor κ_T.

It means practically that a quantity must be found which would take into account by some convenient way the effect of individual parts of wetted perimeter on flow in the channel. Such quantity must have an integral character.

An infinite number of such quantities may be defined without detailed knowledge of hydrodynamic conditions of turbulent flow on the basis of general ideas and requirements. We solved, therefore, this problem in two different ways.

In the first case such a quantity was sought which would have the simplest and logic definition, which would be relatively easy to evaluate and which has been applied at least partly in some earlier dealing turbulent flow.

We have started from the quantity, after a number of unsuccessful attempts, which is used by several authors in their works as a measure for local turbulence and which is essentially interpreted as the mixing in semiempirical theory of turbulent flow in a circular tube.

The dissipation effect of the elements of the channel wall on any perturbation in the region of a point M_0 is inversely proportional in the first approximation to the distance of the point M_0 from the given wall element. As the characteristic distance of the point M_0 from the channel wall may be kept the value:

$$\frac{1}{L(M_0)} = \frac{1}{2}\int_{2\pi}\frac{1}{r(M_0,\phi)}\,d\phi, \tag{4}$$

where $r(M_0,\phi)$ is the distance of the point M_0 from the wall in the direction ϕ. This relationship was used by Buleev [6] in his work. The quantity L, defined by equation (4) is very close to the measure of local turbulence which was derived by Obuchov [7] under the assumption of local similarity of turbulent processes.

The quantity L is local one. The mean value of L in the given cross-section may be considered as the value characterizing the whole channel:

$$\bar{L} = \frac{1}{F}\int_F L\,dF. \tag{5}$$

This quantity is a function both of the channel shape and its dimension. Therefore we have used as the geometrical criterion characterizing only the channel shape the quantity

$$L^* = \frac{\bar{L}}{\bar{L}_0} \tag{6}$$

which is called integral geometrical criterion. The quantity $\bar{L}_0$ is the value of $\bar{L}$ for the equivalent tube, i.e.

$$\bar{L}_0 = 0{\cdot}0887\,.\,d_h.$$

The calculation method for L for any geometry is described in the Appendix.

Table 1. Values of L^* and κ_L for some channel geometries

Channel geometry		L^*	κ_L
	s/d = 1·0	1·630	0·640
	1·025	1·240	0·854
	1·05	1·095	0·984
	1·1	1·000	1·129
Infinite triangular array of	1·2	0·950	1·249
parallel rods	1·3	0·930	1·310
	1·4	0·927	1·354
	s/d = 1·00	1·580	0·645
	1·025	1·385	0·742
	1·05	1·265	0·828
Infinite square array of	1·10	1·135	0·959
parallel rods	1·20	1·010	1·124
	1·30	0·946	1·229
	1·40	0·915	1·303
	a/b = 0·0	0·939	1·225
	0·1	0·973	1·158
	0·2	1·004	1·095
	0·4	1·045	1·010
Rectangle	0·6	1·065	0·97
	0·8	1·075	0·949
	1·0	1·077	0·943
	d_1/d_2 = 0·1	0·95	1·183
	0·2	0·941	1·204
	0·3	0·935	1·212
Smooth annulus	0·4	0·933	1·217
	0·6	0·932	1·221
	1·0	0·939	1·225
	2α = 4·01	1·228	0·874
	7·96	1·209	0·881
Isosceles triangle	12·00	1·193	0·887
	22·30	1·165	0·902
	38·8	1·141	0·915
	60·00	1·132	0·915

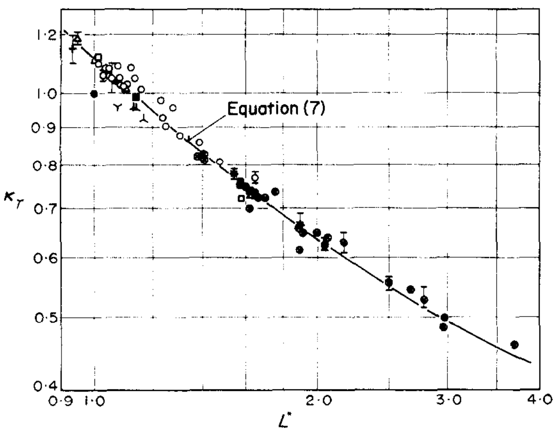

FIG. 1. Correlation between turbulent geometrical factor κ_T and integral geometrical criterion L^*.
●, smooth tube; ○, geometry A [4]; ⊗, geometry B [5]; +, smooth annuli, $d_1/d_2 > 0{\cdot}4$, [1]; Y, square duct [20]; ⅄, triangular ducts [18, 19]; △, infinite triangular array of parallel rods [9–14]; □, infinite square array of parallel rods [15], [17], [25]; ⊠, elements of square array [15]; ✱ flat channels with longitudinal fins [21].

The results obtained from experimental material are plotted as the function $\kappa_T = f(L^*)$ of Fig. 1.

In addition to experimental values of κ_T from the geometries A and B the values of κ_T from some other non-circular channels which were evaluated from papers of other authors were also comprised into these results.

Considering average errors of the results of pressure loss measurements then the experimental points of Fig. 1 have a surprisingly small scattering and the dependence of κ_T on L^* may be estimated as unambiguous. It may be described by the equation:

$$\kappa_T = 0{\cdot}268 + 0{\cdot}842 \, . \, L^{*-1{\cdot}2}. \tag{7}$$

The smooth tube is an exception from this relationship. The expression yields the value $\kappa_T = 1{\cdot}11$ instead of the correct value $\kappa_T = 1$ for this tube. This inconsistency could not be explained satisfactorily till now.

The equation (3) together with the equation (7) represent thus the universal criterion relationship for the friction factor in channels of arbitrary cross-sections with hydraulically smooth walls at a developed turbulent flow.

A more detailed comparison of this relation with experimental results will be still given.

In the second case we started from the assumption of the existence of a connection between the effect of geometry on pressure losses at laminar and turbulent flow.

The friction factor for laminar flow is given by the equation

$$\xi = \frac{C}{Re}. \tag{8}$$

where the constant C is a function of channel shape only. (For circular channel $C = 64$.) By means of the transformation analogous to the transformation (2) we can write formally the universal criterion relationship for ξ for laminar flow in the form

$$\xi = \frac{64}{Re} \, . \, \kappa_L^2 \tag{9}$$

where κ_L is the laminar geometrical factor.

We supposed, that the dependence of κ_T on channel shape can be expressed by means of κ_L.

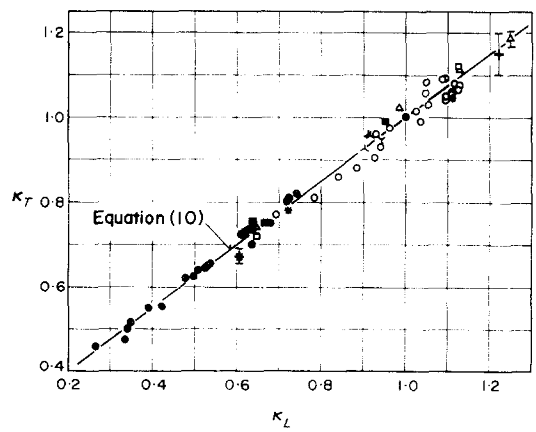

FIG. 2. Correlation between turbulent geometrical factor κ_T and laminar geometrical factor κ_L. Symbols used the same as on Fig. 1.

The results of the treatment of experimental data in the form of the relation $\kappa_T = f(\kappa_L)$ (see Fig. 2) confirmed our assumption. This dependence is relatively unambiguous in the whole range investigated of the channel shapes, i.e.

$$0{\cdot}25 < \kappa_L < 1{\cdot}25$$

or

$$0{\cdot}45 < \kappa_T < 1{\cdot}2$$

and may be expressed by the equation

$$\kappa_T = \frac{1 + 3\kappa_L}{4}. \tag{10}$$

Maximum scattering of experimental data from this dependence is lower than 5 per cent. Contrary to the equation (7) the relationship (10) is valid also for a circular channel.

A comparison is carried out on Figs. 3–11 for the universal relationship (3) with the experimental results on some strongly complexed channel types which are important in practice. They comprise infinite triangular and square lattices of parallel rods, the elements of a square lattice, and excentric annuli. Other cases are given in [1].

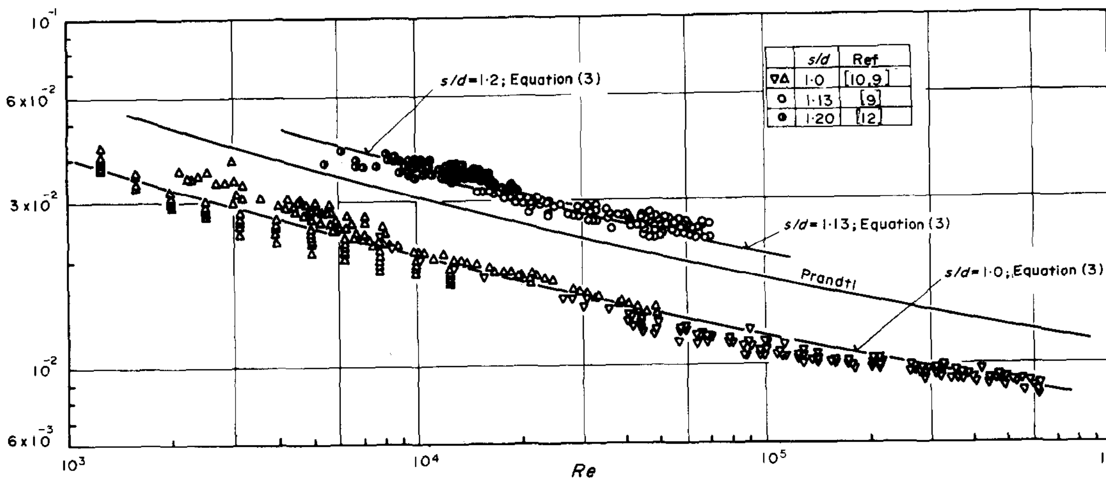

FIG. 3. Comparison of experimental results from infinite triangular arrays with universal criterion relationship (3).

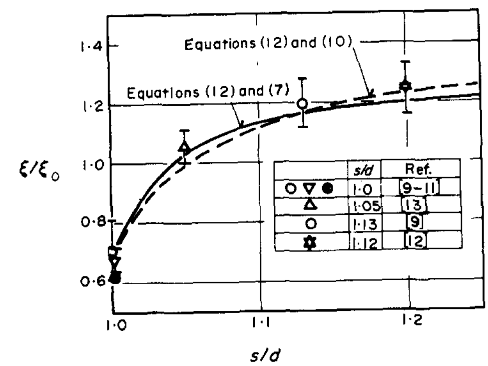

FIG. 4. Dependence of relative friction factor on the rod spacing in infinite triangular array.

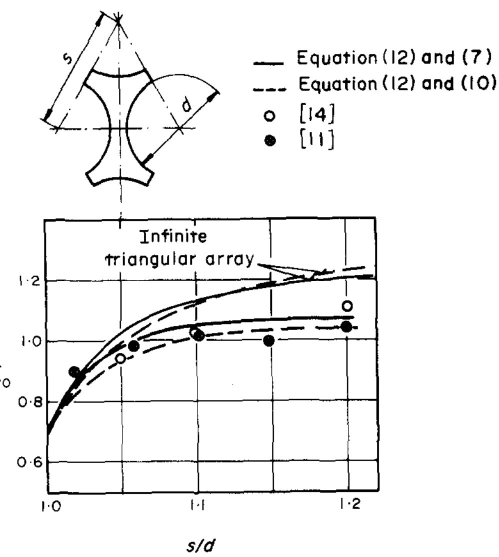

FIG. 5. Relative friction factor for models of triangular array.

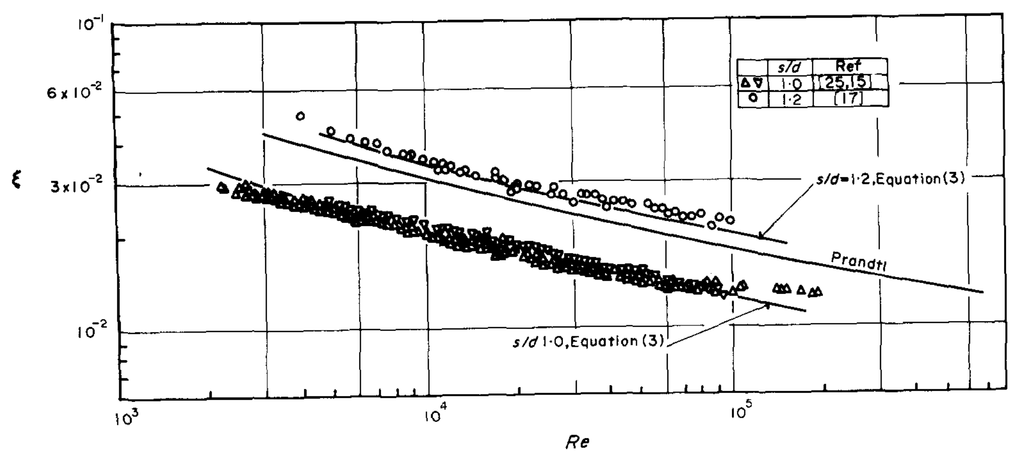

FIG. 6. Comparison of experimental results from infinite square arrays with universal criterion relationship (3).

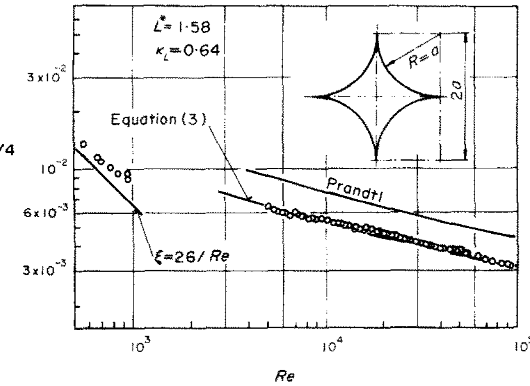

FIG. 7. Comparison of experimental results from square array element channels [15] with universal criterion relationship (3).

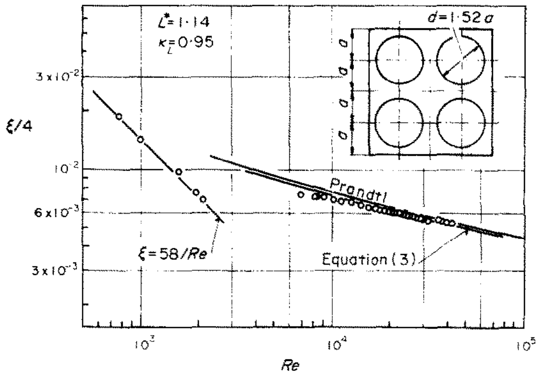

FIG. 10. Comparison of experimental results from square array element channels [15] with universal criterion relationship (3).

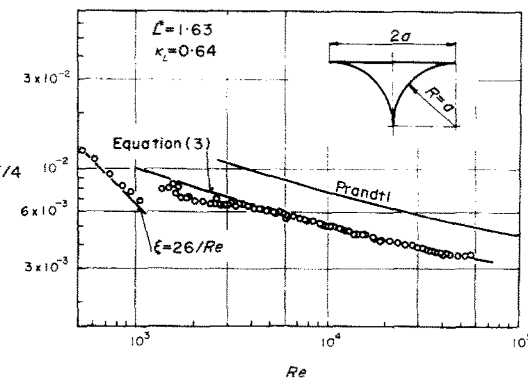

FIG. 8. Comparison of experimental results from square array element channels [15] with universal criterion relationship (3).

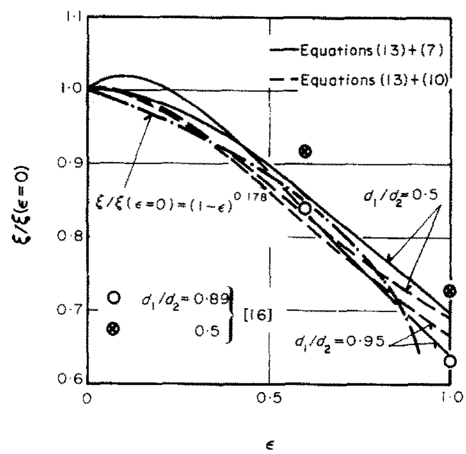

FIG. 11. Dependence of relative friction factor in smooth annulus on its excentricity.

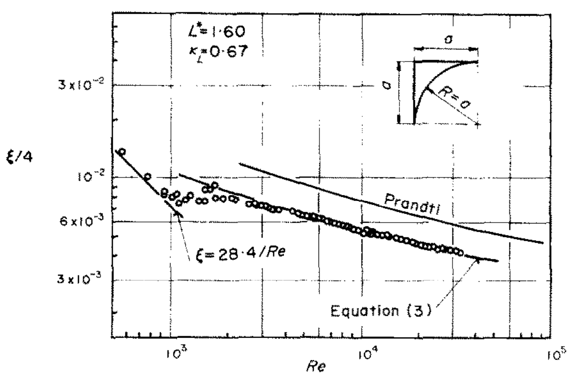

FIG. 9. Comparison of experimental results from square array element channels [15] with universal criterion relationship (3).

A perfect agreement of the relationship (3) with experimental results in the range of several orders of Reynolds number may be seen on Figs. 3 and 6–10. The values of the geometry factor κ_T for the geometries given are practically the same by the relationships (7) and (10). The differences are so small that the application of the relationships (7) and (10) cannot be resolved on the plots.

The universal criterion relationship (3) may be substituted in the region of $Re > 10^4$ with a simpler relationship

$$\xi = 0{\cdot}184 \, . \, \kappa_T^{1{\cdot}2} \, . \, Re^{-0{\cdot}2}. \tag{11}$$

The relative friction factor referred to the friction factor for a smooth tube is then defined by the equation

$$\frac{\xi}{\xi_0} = \kappa_T^{1{\cdot}2}. \tag{12}$$

A comparison of theoretical dependences of the relative friction factor on relative rod pitch with experimental results for an infinite triangular lattice and for the models used to the study of local hydrodynamical conditions in a triangular lattice (see the scheme of Fig. 5) is shown on Figs. 4 and 5. The differences between the application of the relationships (7) or (10) are small in both cases. The scattering of experimental values, however, does not permit to determine which of the two relationships mentioned fits better the real conditions.

The excentricity effect on pressure losses in annular channels may be expressed analogically to the relationship (12) by the equation

$$\frac{\xi}{\xi(\varepsilon=0)} = \left(\frac{\kappa_T}{\kappa_T(\varepsilon=0)}\right)^{1\cdot2}. \tag{13}$$

On Fig. 11 this relationship is compared with the equation

$$\frac{\xi}{\xi(\varepsilon=0)} = (1-\varepsilon)^{0\cdot178}$$

which has been taken from [22] and which according to the authors approximates experimental results with sufficient accuracy neglecting the weak effect of annulus curvature. The points by Tiedt [16] for $d_1/d_2 = 0{\cdot}89$ and 0·5 are also given on the figure. The agreement of the equation (13) with experimental data is again good, only the values by Tiedt for $d_1/d_2 = 0{\cdot}5$ are higher than those by the two relationships given.

HEAT TRANSFER

Heat transfer in non-circular channels is essentially a more complicated problem than heat transfer in a circular channel. It is due to the fact that wall temperature in non-circular channels is not generally constant and its distribution over the heated surface depends both on channel shape and on the distribution of thermal fluxes. On the basis of the universal criterion relationship for the friction factor the universal criterion relationship for the Nusselt number for the case of isothermal flow may be derived using an analogy between momentum and heat transfer. This relationship may be considered as the limit solution of the problem given, which characterizes the effect of channel geometry on heat transfer.

For this purpose we have used essentially the method by Wasan and Wilke [23].

By this method the authors mentioned have derived a criterion relationship for the Nusselt number for a smooth tube, which agrees excellently with experimental results within the range of Prandtl numbers $Pr = 0{\cdot}2$ to 10^4.

The principle of this method is based on the division of the flow cross-section into two regions having different transfer conditions and on an adequate description of the velocity and eddy viscosity ε_w distributions. The first region is in the vicinity of the wall and its boundary is determined by the equation

$$y^+ = 20. \tag{14}$$

The above authors have derived for this region (under the assumption of the validity of the empirical logarithmic velocity distribution in the turbulent core) theoretical relationships for the velocity and eddy viscosity distributions in the form:

$$w^+ = y^+ - 1{\cdot}04\times10^{-4}\times y^{+4} + 3{\cdot}03\times10^{-6}\times y^{+5} \tag{15}$$

$$\frac{\varepsilon_w}{\nu} = \frac{4{\cdot}16\times10^{-4}\times y^{+3} - 15{\cdot}15\times10^{-6}\times y^{+4}}{1-4{\cdot}16\times10^{-4}\times y^{+3} + 15{\cdot}15\times10^{-6}\times y^{+4}}. \tag{16}$$

These expressions agree very well with empirical experiences.

The turbulent core ($y^+ > 20$) represents the second region, in which it is valid that

$$\left.\begin{aligned} \nu &\ll \varepsilon_w \\ \text{and}\quad \frac{\lambda}{\rho\,.\,c_p} &\ll \varepsilon_T \end{aligned}\right\} \tag{17}$$

where ε_T is the eddy conductivity.

It is assumed in both regions

$$\varepsilon_w = \varepsilon_T. \tag{18}$$

This model may also be used for a non-circular channel under the assumption that the distributions of the velocity and the eddy viscosity near the wall are the same as in a circular channel. It may be demonstrated for isothermal flow (when the wall temperature is constant) [24] that between local heat-transfer coefficient and local friction velocity following relation holds

$$\frac{\alpha}{\rho\,.\,c_p} = \frac{\dfrac{v^{*2}}{\bar{w}}}{1+\dfrac{v^*}{\bar{w}}(J_{20}-13)} \tag{19}$$

where

$$J_{20} = \int_0^{20} \frac{dy^+}{\dfrac{1}{Pr}+\dfrac{\varepsilon_w}{\nu}}.$$

The integral J_{20} is a function of the Prandtl number only and its values are tabulated in [23].

The relation (19) was essentially confirmed by the results of our measurements of the distribution of α on a surface with longitudinal fins [24].

For the Nusselt number related to the mean value of the heat-transfer coefficient on a heated surface S_T following equation is then valid:

$$Nu = \frac{\xi}{8}.Re.Pr.\frac{1}{S_T}\int_{S_T}\frac{\tau^*}{1+\sqrt{\left(\dfrac{\xi}{8}\right)}\sqrt{(\tau^*)}(J_{20}-13)}\,dS. \tag{20}$$

If we use again for ξ the relationship (11) then

$$Nu = 0{\cdot}023\,.\,\kappa_T^{1\cdot 2}\,.\,Re^{0\cdot 8}\,.\,Pr\cdot\frac{1}{S_T}\times\int_{S_T}\frac{\tau^*}{1+0{\cdot}152\,.\,\kappa_T^{0\cdot 6}\,.\,Re^{-0\cdot 1}\,.\,\tau^{*1/2}(J_{20}-13)}\,\mathrm{d}S. \quad (21)$$

For the evaluation of this relationship (especially at higher Prandtl numbers) the knowledge of the distribution of τ^* is necessary. For the fluids with $Pr \to 1$ the expression (21) may be simplified.

$$Nu = 0{\cdot}023\,.\,\kappa_T^{1\cdot 2}\,.\,Re^{0\cdot 8}\,.\,Pr\,.\frac{1}{S_T}\times\int_{S_T}[\tau^*-0{\cdot}152\,.\,\kappa_T^{0\cdot 6}\,.\,Re^{-0\cdot 1}\,.\,\tau^{*1\cdot 5}(J_{20}-13)]\,\mathrm{d}S. \quad (22)$$

We may write then in the first approximation

$$Nu = 0{\cdot}023\,.\,\phi\,.\,Re^{0\cdot 8}\times Pr[1-0{\cdot}152\,.\,\phi^{0\cdot 5}\,.\,Re^{-0\cdot 1}(J_{20}-13)] \quad (23)$$

where

$$\phi = \kappa_T^{1\cdot 2}\frac{\bar{\tau}_T}{\bar{\tau}} \quad (24)$$

and $\bar{\tau}_T$ is the mean value of the shear stress on the heated surface S_T.

The condition of isothermal flow was not met in the investigation of heat transfer in the channels of the geometries A and B. A comparison of our experimental results with the expression (23) may be carried out only at some complementary assumptions.

The conditions of the geometry B approached mostly the assumptions under which the expression (23) was derived. Thermal output along the model length was constant and air was selected as cooling medium ($J_{20} = 9{\cdot}7$). Temperature of the wetted perimeter was practically constant. Operational conditions were selected so that the temperature factor ψ (defined as the ratio of absolute temperatures of the wall and fluid) was the same as possible for all models at all operational conditions. Its mean value from all measurements was $\bar{\psi} = 1{\cdot}27$.

Let us assume in the first approximation that the effect of the temperature factor in non-circular channels may be expressed by the Kutateladze relationship, which has been derived for a smooth tube, in the form

$$Nu = Nu_{\psi=1}\left(\frac{2}{(\sqrt{\psi})+1}\right)^2 \doteq Nu_{\psi=1}\,.\,\psi^{-0\cdot 55}. \quad (25)$$

The effect of the variation of the Reynolds number in the range of two orders of magnitude on the second term of the right side of the equation (23) may be neglected. To compare experimental results with the relation (23) it is then sufficient to evaluate the dependence $Nu = f(\phi)$. At the selected $Re = 5\times 10^4$ following equation should then be valid for our experiments

$$Nu = 81{\cdot}1\,.\,\phi(1+0{\cdot}17\,.\sqrt{\phi}) \quad (26)$$

where

$$\phi = \kappa_T^{1\cdot 2}$$

for the geometry B.

The results of the experimental data are shown on Figs. 12 and 13. Having considered the attainable accuracy of thermokinetical experiments it may be

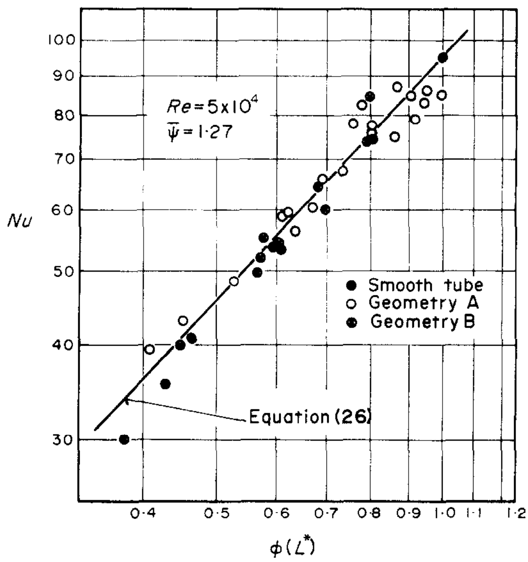

FIG. 12. Relation $Nu = f(\phi)$ for geometry A and B when $\phi = \phi(L^*)$.

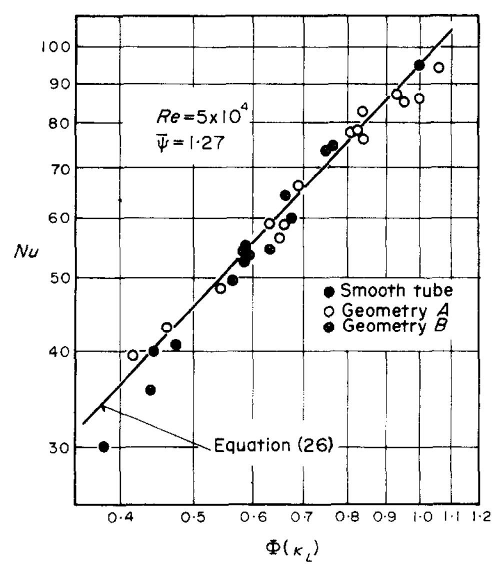

FIG. 13. Relation $Nu = f(\phi)$ for geometry A and B when $\phi = \phi(\kappa_L)$.

stated that the results from the geometry B agree very well with the relationship (26) both with the use of the relation (7) and (10) for determining κ_T. [In the first case ϕ is denoted as $\phi(L^*)$, in the second case as $\phi(\kappa_L)$.] The results from smooth circular channels also satisfy very well the relationship (26).

The case of the geometry A is substantially more complex. The flow cross-section is represented by a doubly connected region the heated surface being only the surface of the finned kernel. Temperature on the perimeter of the finned kernel was practically constant. Kernel temperature and the temperature of the external tube, however, differred essentially. Otherwise the experimental conditions were the same as in the geometry B. Though the experimental conditions differed from those of isothermal flow we started by treating experimental data from the assumption that even in this case the equation (26) holds. In the definition equation (24) we put

$$\frac{\bar{\tau}_T}{\bar{\tau}} \doteq \left(\frac{d_T}{d_h}\right)^n. \tag{27}$$

This equation may be understood as a condition of force equilibrium. The quantity d_T has the meaning of the hydraulic diameter related to the part of the flow cross-section, which was bounded by heated surface on one side and by the line of maximum values $L(M_0)$ on the other side [in the case when κ_T is defined by the equation (7)] or by the line of maximum velocities [when using the relation (10)]. The exponent n was taken as a constant in the first approximation and evaluated from experiment. In the first case $n = 1{\cdot}42$, in the second $n = 1{\cdot}73$.

The results obtained are shown on Figs. 12 and 13. Also in this case the results yield a good idea about the effect of the channel shape on heat transfer.

For completeness it is necessary to mention, however, that by some experimental results [1] the exponent of the temperature factor in the equation (25) will also depend probably on the channel shape.

CONCLUSIONS

The universal criterion relationship (3) together with the equation (7) or (10) describes the dependences of the coefficient of friction losses on Reynolds number and on channel geometry in the channels of any cross-section with hydraulically smooth walls. This was confirmed by the comparison with all available data from literature as far as these data were obtained really in conditions of a fully developed turbulent flow.

The universal criterion relationship for Nusselt number (20) for isothermal flow represents the limit and characteristic solution of the problem of heat transfer in non-circular channels. The relationship (23), which is a simplification of the relationship (20) for the fluid with the Prandtl number approaching to one, agrees well with experimental results. Both relationships enable a relatively accurate evaluation of the influence of channel shape on heat transfer.

REFERENCES

1. J. Malák, Pressure losses and heat transfer in non-circular channels with hydraulically smooth walls (transl.), Report UJV 2891 R (1972).
2. J. Ort, Experimental research of turbulent flow and heat transfer in annuli (transl.), Report SVÚSS 64-08004 (1964).
3. F. J. Quirrenbach, Wärmeübergang bei turbulenter Strömung in Ringspalten, *Allg. Wärmetech.* **9**(13) 271–276 (1960).
4. J. Malák, M. Řezníček, J. Červenka and J. Hejna, Hydrodynamics and thermokinetics in annuli with longitudinal fins (transl.), Report UJV 2187 R,T (1969).
5. J. Červenka, J. Malák, M. Řezníček and J. Stibor, Hydrodynamics and thermokinetics in circular tubes with longitudinal fins (transl.), Report UJV 2501 R,T (1971).
6. N. I. Buleev, Theoretical model for turbulent transfer in three-dimensional fluid flow (transl.), *Coll. "Heat Transfer"* (transl.), Izd. A. Sci. USSR, Moscow (1962).
7. A. N. Obuchov, Turbulence scale distribution in flows with arbitrary cross-section (transl.), *Prikl. Mat. Mech.* **6**(2–3), 209–219 (1942).
8. J. O. Hinze, *Turbulence.* McGraw-Hill, New York (1959).
9. V. I. Subbotin, P. A. Ushakov and B. N. Gabrianovitch, Hydraulic resistance to the flow along a bundle of rods (transl.), *Atomn. Energ.* **9**(4), 308–310 (1960).
10. J. V. Levtchenko, V. I. Subbotin, P. A. Ushakov and A. V. Sheynina, Velocity distribution in a narrow triangular array of parallel rods (transl.), *Coll. "Liquid metals"* (transl.). Atomizdat, Moscow (1967).
11. W. Eifler and R. Nijsing, Experimental investigation of velocity distribution and flow resistance in a traingular array of parallel rods, *Nucl. Engng Design* **5**, 22–24 (1967).
12. E. V. Firsova, Investigation of heat transfer and hydraulic resistance for the flow along a bundle of rods. (transl.), *Inzh. Fiz. Zh.* **6**(5), 17–22 (1963).
13. A. V. Sheynina, Hydraulic resistance to the flow along a bundle of rods (transl.), *Coll. "Liquid metals"* (transl.). Atomizdat, Moskva (1967).
14. V. I. Subbotin, P. A. Ushakov, J. D. Levchenko and A. M. Aleksandrov, Velocity field of turbulent flow along a rod bundle (transl.), Reactor Heat Transfer Conference, Babylon, Czechoslovakia (1970).
15. D. J. Gunn and C. W. W. Darling, Fluid flow and energy losses in non-circular conduits, *Trans. Instn Chem. Engrs* **41**, 163–173 (1963).
16. W. Tiedt, Berechnung des laminaren und turbulenten Reibungswiderstandes konzentrischer und exzentrischer Ringspalte, Dissertation, Darmstadt (1966).
17. B. W. Le Tourneau and R. E. Brimbal, J. E. Zerbe, Pressure drop for parallel flow through rod bundles, *Trans. Am. Soc. Mech. Engrs* **79**(8), 1751 (1957).
18. L. Schiller, Über den Stromungswiderstand von Rohren verschiedenen Querschnittes und Rauhigkeitsgrades, *Z. Angew. Math. Mech.* **3**, 2 (1923).
19. L. W. Carlson and T. F. Irvine, Jr., Fully developed pressure drop in triangular shaped ducts, *J. Heat Transfer* **83C**, 441–444 (1961).

20. E. Brundrett and P. R. Burroughs, The temperature inner-law and heat transfer for turbulent air flow in a vertical square duct, *Int. J. Heat Mass Transfer* **10**(9), 1133–1142 (1967).
21. J. Malák, J. Hejna, J. Červenka and F. Mantlik, Experimental research of local hydrodynamical conditions in flat channels with longitudinal fins—II (transl.), Report UJV 3153/R (1973).
22. W.-S. Yu and O. E. Dwyer, Heat transfer to liquid metals flowing turbulently in excentric annuli—I. *Nucl. Sci. Engng* **24**, 105–117 (1966).
23. D. T. Wasan and Ch. R. Wilke, Turbulent exchange of momentum, mass, and heat between fluid streams and pipe wall, *Int. J. Heat Mass Transfer* 7(1), 87–94 (1964).
24. J. Malák, J. Hejna and J. Červenka, Experimental research of heat transfer coefficient distribution on a plate with longitudinal fins by means of analogy between heat and mass transfer—II (transl.), Report UJV 3154/R (1973).
25. P. A. Ushakov, V. I. Subbotin, B. N. Gabrianovich, V. D. Talamov and I. P. Sviridenko, Heat transfer and hydraulic resistance in narrow square array of parallel rods (transl.), *Atomn. Energ.* **13**(2), 162–169 (1962).
26. J. Schmid, Solution of steady temperature fields by means of finite elements method (transl.), Report UJV 2989 R (1973).

APPENDIX

Calculation of $L(M_0)$ and of the Laminar Velocity Field

The calculation of all quantities was accomplished on the GIER computer. A program was compiled to calculate the values L, which makes possible the calculations for any two-dimensional region. The boundary of the region investigated is approximated there with any number of line sections and circular arcs. The integral in the equation (4) is evaluated by numerical integration over the region limited by symmetry axes. In the case of the beam and axis intersection the condition of reflection is applied and thus the same effect is obtained as for the whole region. A quicker calculation method starts from the fact that in the case of the region limited by linear sections the integral (see Fig. 14a)

$$J(M_0) = \int_{\phi_1}^{\phi_2} \frac{1}{r(M_0,\phi)}\,\mathrm{d}\phi$$

has the solution

$$J(M_0) = \frac{\sin\alpha_2 - \sin\alpha_1}{l}.$$

Dividing the boundary of the region investigated (which can be considerably complex and exhibit self-screening) to i line sections the value L may be calculated from the relationship

$$L(M_0) = \frac{2}{\sum_i J_i(M_0)}.$$

This method is three to five times quicker than the original method. The mean values $\bar{L}$ were calculated by Gauss integration formula.

Laminar flow in the channels of non-circular cross-sections was solved numerically by the mesh method [26]. A set of programs which were compiled to this purpose make possible the calculation for any two-dimensional region. The difference equations are solved there by the Peaceman–Rachford iteration method. For the calculation in strongly complicated channels the finite elements method appeared as more convenient with respect to the accuracy of calculations and computer utilization. The method and the programs are described in details in [27].

The Calculation of d_T for the Doubly Connected Region

The line of maximum values of L or the line of maximum velocities at laminar flow was determined by following way (see Fig. 14b): At first, the points A and B with maximum L value (or w) were determined on symmetry axes. The trajectory orthogonal to the level curves $L = \text{const.}$ (or $w = \text{const.}$), which is the projection of the slope curve passing on the top of the plane $L(x, y)$ [or $w(x, y)$] must pass through these points with respect to symmetry conditions. In the calculation we start from the point A the initial direction of the required trajectory being $\psi = \pi/2$. At a certain distance ΔS from the point A the values L (or w) for the angles $\psi - \Delta\psi$, ψ, $\psi + \Delta\psi$ were determined. A parabole of the second degree passing through these three points was constructed. Its maximum is on the wanted trajectory. Analogically we proceed up to the point B. Then the value d_T was determined from the area limited by this trajectory and by the heated surface.

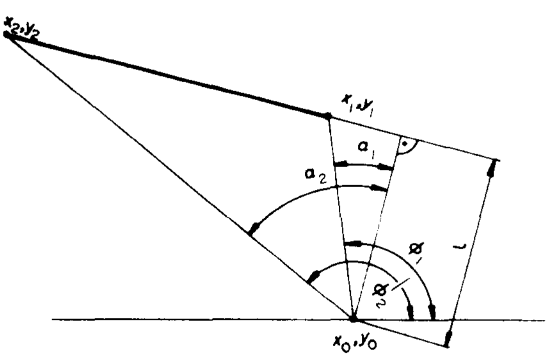

FIG. 14a.

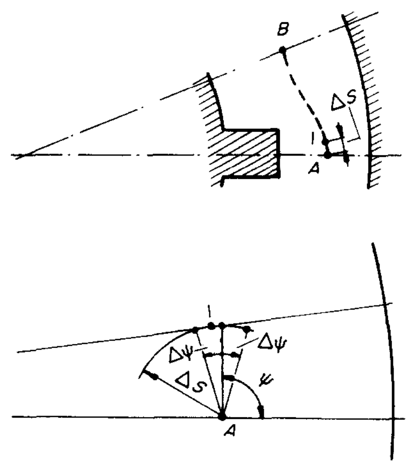

FIG. 14b.

PERTE DE CHARGE ET TRANSFERT THERMIQUE DANS LES CANAUX NON CIRCULAIRES A PAROIS HYDRAULIQUEMENT LISSES

Résumé—Cet article traite de la détermination de l'effet de la géométrie du canal sur la perte de pression et sur le transfert thermique dans l'écoulement turbulent établi dans ces canaux non circulaires, à parois lisses. On donne l'expression du coefficient de frottement en tenant compte de l'effet de la géométrie par un facteur géométrique χ_r. La relation empirique entre le facteur géométrique χ_r et la forme du canal est déterminée de deux façons: par le critère intégral L et par le facteur laminaire χ_L.

Utilisant une analogie entre transferts de chaleur et de quantité de mouvement, l'expression du nombre de Nusselt est déduite du critère universel ξ.

On considère les résultats expérimentaux pour évaluer et tester les deux expressions universelles. Dans la plupart des cas, on trouve un excellent accord avec les résultats expérimentaux.

DRUCKVERLUST UND WÄRMEÜBERTRAGUNG IN NICHT-KREISFÖRMIGEN KANÄLEN MIT HYDRAULISCH GLATTEN WÄNDEN

Zusammenfassung—Der Einfluß von geometrischen Verhältnissen auf den Reibungsdruckverlust und auf die Wärmeübertragung in der ausgebildeten turbulenten Strömung in nicht-kreisförmigen Kanälen mit hydraulisch glatten Wänden wird ermittelt. Eine allgemeine dimensionslose Beziehung für den Reibungsfaktor wird aufgestellt, in der die Wirkung der geometrischen Verhältnisse in Form eines Geometriefaktors κ_r berücksichtigt wird. Die empirisch aufgestellte Beziehung zwischen dem Geometriefaktor κ_r und der Form des Kanals wird auf zwei Arten ausgewertet: durch das integrale Geometriekriterium L^* und durch den laminaren Geometriefaktor κ_L. Die allgemeine dimensionslose Beziehung für die Nusselt-Zahl in isothermen Strömungen wird unter Verwendung einer Analogie zwischen Impulsaustausch und Wärmeübertragung aus der allgemeinen Kennzahlen-Gleichung für ξ abgeleitet.

Umfangreiche experimentelle Ergebnisse wurden für die Bewertung und Überprüfung beider allgemeinen Beziehungen verwendet. Die Übereinstimmung zwischen experimentellen Werten und Ergebnissen aus der Kennzahlen-Gleichung war in den meisten Fällen ausgezeichnet.

ПОТЕРИ ДАВЛЕНИЯ И ТЕПЛООБМЕН В КАНАЛАХ НЕКРУГЛОГО СЕЧЕНИЯ С ГИДРАВЛИЧЕСКИ ЖИДКИМИ СТЕНКАМИ

Аннотация — В статье исследуется влияние геометрии канала на потери давления и теплообмен в развитом турбулентном течении в каналах некруглого сечения с гидравлически гладкими стенками. Сформулировано универсальное безразмерное соотношение для коэффициента трения, в котором влияние геометрии учитывается с помощью форм-фактора κ_T. Эмпирическое соотношение для κ_T и формы канала рассчитывается двумя способами: с помощью интегрального геометрического критерия L^* и ламинарного форм-фактора κ_L.

Используя аналогию между переносом импульса и тепла, универсальное безразмерное соотношение для числа Нуссельта в изотермическом течении выводится из универсального критериального соотношения для ξ.

Для оценки и проверки этих двух универсальных соотношений проведена обработка обширного экспериментального материала. В большинстве случаев сравнение показало прекрасное совпадение экспериментальных данных с универсальным критериальным соотношением.